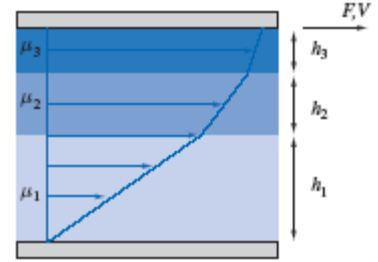


## Problem 2.57

[Difficulty: 2]

**2.57** Fluids of viscosities  $\mu_1 = 0.15 \text{ N} \cdot \text{s}/\text{m}^2$ ,  $\mu_2 = 0.5 \text{ N} \cdot \text{s}/\text{m}^2$ , and  $\mu_3 = 0.2 \text{ N} \cdot \text{s}/\text{m}^2$  are contained between two plates (each plate is  $1 \text{ m}^2$  in area). The thicknesses are  $h_1 = 0.5 \text{ mm}$ ,  $h_2 = 0.25 \text{ mm}$ , and  $h_3 = 0.2 \text{ mm}$ , respectively. Find the steady speed  $V$  of the upper plate and the velocities at the two interfaces due to a force  $F = 100 \text{ N}$ . Plot the velocity distribution.



**Given:** Flow of three fluids between two plates

**Find:** Upper plate velocity; Interface velocities; plot velocity distribution

**Solution:**

The shear stress is the same throughout (the velocity gradients are linear, and the stresses in the fluids at the interfaces must be equal and opposite).

Given data	$F = 100 \cdot \text{N}$	$h_1 = 0.5 \cdot \text{mm}$	$h_2 = 0.25 \cdot \text{mm}$	$h_3 = 0.2 \cdot \text{mm}$
	$A = 1 \cdot \text{m}^2$	$\mu_1 = 0.15 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$	$\mu_2 = 0.5 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$	$\mu_3 = 0.2 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$

The (constant) stress is	$\tau = \frac{F}{A}$	$\tau = 100 \text{ Pa}$
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For each fluid	$\tau = \mu \cdot \frac{\Delta V}{\Delta y}$ or	$\Delta V = \frac{\tau \cdot \Delta y}{\mu}$	where $\Delta V$ is the overall change in velocity over distance $\Delta y$
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Hence	$V_{12} = \frac{\tau \cdot h_1}{\mu_1}$	$V_{12} = 0.333 \frac{\text{m}}{\text{s}}$	where $V_{12}$ is the velocity at the 1 - 2 interface
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Hence	$V_{23} = \frac{\tau \cdot h_2}{\mu_2} + V_{12}$	$V_{23} = 0.383 \frac{\text{m}}{\text{s}}$	where $V_{23}$ is the velocity at the 2 - 3 interface
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Hence	$V = \frac{\tau \cdot h_3}{\mu_3} + V_{23}$	$V = 0.483 \frac{\text{m}}{\text{s}}$	where $V$ is the velocity at the upper plate
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